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EE-560: Probability, Random Variables, and Stochastic Processes
Fall 2012

Faculty Information:

Instructor: Professor Mohsen Kavehrad
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Teaching Assistant:
Grader for this course is **TBD**, his office hours are TBD, ROOM 111 in EE-EAST and his e-mail is:

Required Text:
Papoulis A. and Pillai S., *Probability, Random Variables and Stochastic Processes*, ISBN: 2002 9780073660110, 4th Ed., McGraw-Hill, NY, 2002.

Grading Policy:

Written homework	20%
Exam I ….., Thursday evening	20%
Exam II …., Thursday evening	25%
Final Exam	35%

Homework and projects will generally be due on Tuesday or Thursday at the start of the lecture.

Prerequisites:
EE-350, STAT-418

References:

1. Course Notes
2. William Feller, An Introduction to Probability Theory and its Applications, Volume-I, Third Edition, Wiley & Sons, 1990.
3. V.S. Pugachev, Probability Theory and Mathematical Statistics for Engineers, Pergamon Press Ltd., 1984.
4. A. Leon-Garcia, Probability and Random Processes for Electrical Engineering, Addison-Wesley, 2nd Edition, 1994.
5. H. Stark and J. W. Woods, Probability and Random Processes with Applications to Signal Processing 3rd Ed., Prentice-Hall, Englewood Cliffs, NJ, 2001.
6. P. Z. Peebles, Probability, Random Variables, and Random Signal Principles, 3rd. Ed., McGraw-Hill, 1993.

Course Description
Review of probability theory and random variables; mathematical description of random signals; linear systems response.

Course Objectives

This course deals with the mathematical theory that was developed in the last century to describe and analyze the variety of "random" phenomena that occur in the various branches of electrical engineering and other fields. This theory developed after a long period during which "chance phenomena" had been dealt with using a variety of ad hoc approaches, many of which were inconsistent and lead to mathematicians disparaging the whole field claiming that "probability theory" was not a proper mathematical subject. This situation was altered by a Soviet school of mathematicians led by A.N. Kolmogorov, which, in the 1930's, finally established a proper mathematical basis for a theory of probability.

The theory of probability is intended to reflect a particular view we sometimes adopt to explain how the world behaves. Underlying this theory are two basic notions: the notion of the outcome of an experiment being random, and the concept of the probability of a particular type of outcome. Recent physics has seen the evolution of a nondeterministic theory of quantum mechanics, which has seemed able to accurately predict many observed phenomena the deterministic theories could not. Notwithstanding the success of this nondeterministic theory in applications, not everyone is convinced of the fundamental validity of random models for our world (e.g., we have Einstein's often quoted view in reference to his reluctance to embrace quantum mechanics as a fundamental description of reality, that "God does not play dice" ... to which Niels Bohr's retort "Who are you to tell God what to do"). The debate on the fundamental nature of the universe is a metaphysical debate - seemingly without resolution on which we will not dwell. Instead, we shall simply agree to accept this model as our (ideal) concept of what we refer to as a random event and set aside the question of whether or not it is as an absolutely accurate description of reality.

A justification of random models in practice is possible on pragmatic grounds quite apart from accepting the existence of "ideally random" phenomena. In the real (causal) world, we are often faced with events which we might regard as deterministic, but for which we do not know some or all of the determining factors, e.g., weather prediction or stock market fluctuations. In such cases, we frequently model the event as random although we fully accept that it would not be so according to our ideal notion above. Therefore, we admit ignorance and use probabilistic tools to measure likelihood of our predictions.

Before we begin the mathematical discussion, we examine these concepts to understand how the mathematical theory is supposed to relate to the physical world and application.

The text used in this course is a classic one. It is neither a handbook, nor a tutorial book. It contains a logical sequence of topics presented in a smooth and continuous manner.

Tentative Schedule

I. INTRODUCTION

1.1 THE MEANING OF "RANDOM"
1.2 THE CONCEPT OF PROBABILITY
1.3 PROLOGUE

II. PROBABILITY THEORY

2.1 SET THEORY
2.1.1 Basic Notions
2.1.2 Extended Union and Intersection
2.2 PROBABILITY SPACES
2.2.1 Event Space Properties (Axioms)
2.2.2 Probability Measure Properties (Axioms)
2.2.3 The Borel Algebra
2.3 THE CONTINUITY THEOREM OF PROBABILITY
2.3.1 The Borel-Cantelli Lemma
2.4 CONDITIONAL PROBABILITY
2.4.1 Bayes' Rule
2.4.2 Independence
2.5 REPEATED TRIALS
2.5.1 Generalization
2.5.2 Bernoulli Trials (Experiments)
2.5.3 Generalized Bernoulli Trials

--- [2.5 weeks]

III. RANDOM VARIABLES
3.1 THE PROBABILITY DISTRIBUTION FUNCTION
3.1.1 Properties of the Distribution Function
3.1.2 The Existence Theorem
3.2 THE PROBABILITY DENSITY FUNCTION
3.2.1 Properties of a Probability Density Function
3.2.2 Extended Notion of a Probability Density Function
3.3 CLASSICAL DISTRIBUTIONS
3.3.1 Discrete Distributions
3.3.2 Continuous Distributions
3.4 CONDITIONAL DISTRIBUTION FUNCTIONS AND DENSITY FUNCTIONS

IV. FUNCTIONS OF A RANDOM VARIABLE
4.1 TRANSFORMATIONS OF RANDOM VARIABLES
4.2 MATHEMATICAL EXPECTATION
4.2.1 The Expectation of $Y = g(X)$
4.2.2 The Linearity of Expectation
4.2.3 Conditional Expectation
4.3 MOMENTS
4.3.1 Variance
4.3.2 General Moments
4.3.3 The Chebyshev Inequality
4.4 COMPLEX RANDOM VARIABLES AND CHARACTERISTIC FUNCTIONS
4.4.1 The Moment Theorem
4.5 GENERATING FUNCTIONS
-- [3 weeks]

V. TWO AND MORE RANDOM VARIABLES
5.1 JOINT DISTRIBUTION AND DENSITY FUNCTIONS
5.1.1 Extensions of the Notion of Conditional Distributions and Densities to Jointly Continuous Random Variables
5.1.2 Independent Random Variables
5.2 FUNCTIONS OF TWO RANDOM VARIABLES
5.2.1 One Function of Two Random Variables
5.2.2 Two Functions of Two Random Variables
5.2.3 Expectation
5.2.4 Conditional Expectation
5.3 JOINT MOMENTS
5.3.1 Basic Definitions
5.3.2 Joint Moments and Linear Transformations
5.4 JOINT CHARACTERISTIC FUNCTIONS
5.5 JOINTLY GAUSSIAN RANDOM VARIABLES
5.6 COMPLEX RANDOM VARIABLES
5.7 THE GEOMETRY OF RANDOM VARIABLES
5.8 Linear Estimation

VI. SEQUENCES OF RANDOM VARIABLES
6.1 CONVERGENCE CONCEPTS
6.2 THE LAWS OF LARGE NUMBERS
6.3 THE CENTRAL LIMIT THEOREM
-- [4 weeks]

VII. STOCHASTIC PROCESSES: INTRODUCTION
7.1 BASIC CONCEPTS

7.2 CLASSICAL RANDOM PROCESSES (EXAMPLES: Poisson, Markov, ….)
7.3 RANDOM PROCESS CHARACTERIZATION
  7.3.1 Complete Characterizations
  7.3.2 Stationary Processes
  7.3.3 Results for Classical Random Processes
7.4 GAUSSIAN RANDOM PROCESSES
7.5 ERGODICITY

VIII. SECOND-ORDER PROCESSES
8.1 TRANSFORMATIONS OF A RANDOM PROCESS
8.2 THE POWER SPECTRUM
  8.2.1 General Properties
  8.2.2 White Noise
8.3 RANDOM PROCESS REPRESENTATIONS
  8.3.1 Rice's Representation Theorem
  8.3.2 Karhunen-Lo`eve Expansions

IX. APPLICATIONS OF RANDOM PROCESSES
--- [5.5 weeks]

Course Requirements
Homework will be assigned on a weekly base and solutions are due the following week at the beginning of the lecture. All questions assigned will be marked. Solution methods must be clearly explained! Late assignments will not be accepted. Solutions to the problem set will be provided (usually immediately following the assignment due date).
It is anticipated that problems will be discussed among students and this is encouraged. However, submitted solutions must be written independently reflecting your understanding. Plagiarism will earn marks of zero for all concerned as well as possibility of further sanctions.

Attendance Policy
Students are expected to attend all the classes and are encouraged to participate during discussion in class. Also, attending the problem solving sessions on Fridays is strongly recommended.

Academic Integrity Policy
Academic dishonesty includes any form of cheating that gives unfair advantage to a student. Any student charged with academic dishonesty must see the instructor immediately following the infraction to discuss the consequences. As described in Faculty Senate Policy 49-20, the sanction could range from a zero score on the exam or HW to disciplinary expulsion.

Examination Policy
Two mid-semester exams will be given in the evening on October 11, 2012 and November 15, 2012 starting at 6:30 pm in Room: 225 EE-WEST.
The tentative date for the final exam is December 2012. Exams will be closed book. A specified number of note sheets will be announced prior to exams.